Lecture 18: Encrypting Long Messages

Objective

- Earlier, we saw that the length of the secret key in a one-time pad has to be at least the length of the message being encrypted
- Our objective in this lecture is to use smaller secret keys to encrypt longer messages (that is, secure against computationally bounded adversaries)

Recall

- Suppose $f: \{0,1\}^{2n} \to \{0,1\}^{2n}$ is a one-way permutation (OWP)
- Then, we had see that the function $G: \{0,1\}^n \times \{0,1\}^n \to \{0,1\}^{2n+1}$ defined by

$$G(r,x) = (r, f(x), \langle r, x \rangle)$$

  is a one-bit extension PRG
- Let us represent $f^i(x)$ as a short-hand for $\overbrace{f(\cdots f(f}^{i\text{-times}}(x))\cdots)$. $f^0(x)$ shall represent x.
- By iterating the construction, we observed that we could create a stream of pseudorandom bits by computing $b_i(r,x) = \left\langle r, f^i(x) \right\rangle$ (Note that, if we already have $f^i(x)$ stored, then we can efficiently compute $f^{i+1}(x)$ from it)
- So, the idea is to encrypt long messages where the i-th bit of the message is masked with the bit $b_i(r,x)$

Encrypting Long Messages

- Without loss of generality, we assume that our objective is to encrypt a stream of bits $(m_0, m_1, \dots)$
- Gen(): Return $\mathsf{sk} = (r, x) \xleftarrow{\$} \{0,1\}^{2n}$, where $r, x \in \{0,1\}^n$
- Alice and Bob shall store their state variables: state_A and state_B. Initially, we have $\mathsf{state}_A = \mathsf{state}_B = x$
- $\mathsf{Enc}_{sk, \mathsf{state}_A}(m_i)$: $c_i = m_i \oplus \langle r, \mathsf{state}_A \rangle$, and update $\mathsf{state}_A = f(\mathsf{state}_A)$, where $\mathsf{sk} = (r, x)$
- $\mathsf{Dec}_{sk, \mathsf{state}_B}(\widetilde{c}_i) = \widetilde{m}_i = \widetilde{c}_i \oplus \langle r, \mathsf{state}_B \rangle$, and update $\mathsf{state}_B = f(\mathsf{state}_B)$, where $\mathsf{sk} = (r, x)$
- Note that the i-th bit is encrypted with $b_i(r, x)$ and is also decrypted with $b_i(r, x)$. So, the correctness holds. This correctness guarantee holds as long as the order of the encryptions and the decryptions remain identical.
- Note that each bit $b_i(r, x)$ is uniform and independent of all previous bits (for computationally bounded adversaries). So, the scheme is secure against all computationally bounded adversaries